

User Guide to “SPC Filter 2016-06-29.xlsx”
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The Excel VBA program “SPC Filter 2016-06-29.xlsx” compares a first-order (FO) filter to the statistical process control (SPC) filter.

The user enters choices in the green shaded cells in the “Sheet1” worksheet. The entry table is:

Table 1. User input.

N realizations =	100	trigger =	2
Initialization (n/p) =	n	x1 =	8
pause time =	0.1	sigma1 =	1
known variance =	N	change =	50
ramp/step =	S	x2 =	5
		sigma2 =	0.5
		f of lambda =	0.053

In the right hand column, “trigger” is the SPC “sigma level” to trigger a change. The value of 2 represents the 2-sigma level, or roughly 95% confidence. I think values of 1.5 to 3 are reasonable choices. The values x1 and x2 are the initial and final true values of the PV, and the point of change is sampling number 50. The signal values for the noise on x1 and x2 in the before and after change point are 1 and 0.5. The first-order filter (f of f) coefficient value that makes the FO filter equivalent to the SPC filter with a trigger of 2, is 0.053.

Here is a table of equivalence values for trigger and lambda. Equivalence is based on the root-mean-square (rms) deviation of the filtered value from the true value during an extensive stationary period (constant x-value and constant sigma-value) when the filters are both initialized with the true value.

Table 2. Filter factor equivalence values.

SPC trigger	f of f lambda
1.5	0.163
1.75	0.089
2	0.053
2.25	0.032
2.5	0.022
2.75	0.0145
3	0.00905
3.5	0.0055

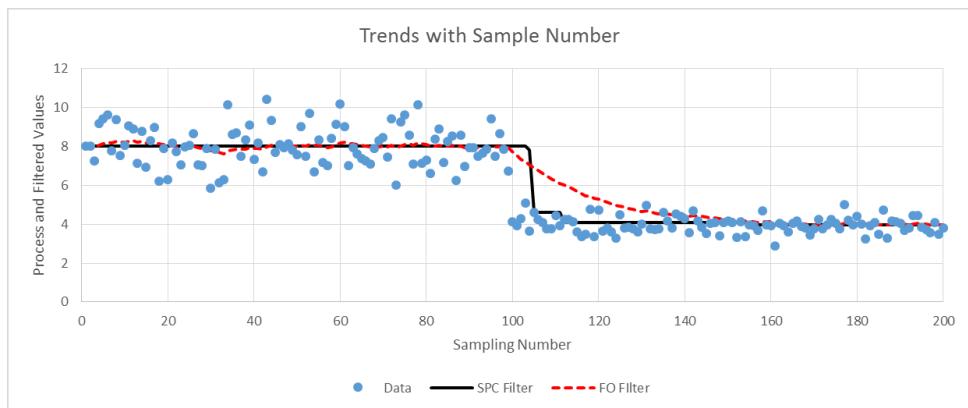
The left hand column of Table 1 is for test choices. In “N realizations” choose the number of trials. I find that to get consistent average data, 1,000 to 5,000 trials are needed. In “Initialization” the letter “n”

represents nominal (the FO filter is initialized with the first measurement, and the SPC filter is initialized with zero. An entry value of "p" indicates perfect initialization with the true x and sigma values, which is not realizable in practice, but can simplify the interpretation of simulation results. "pause_time" is your choice of an execution pause between realizations, it slows the program, but permits visual observation of the results at each iteration. I think values of 0.1 to 1 (second) are good. A zero means that the computer only periodically update the display.

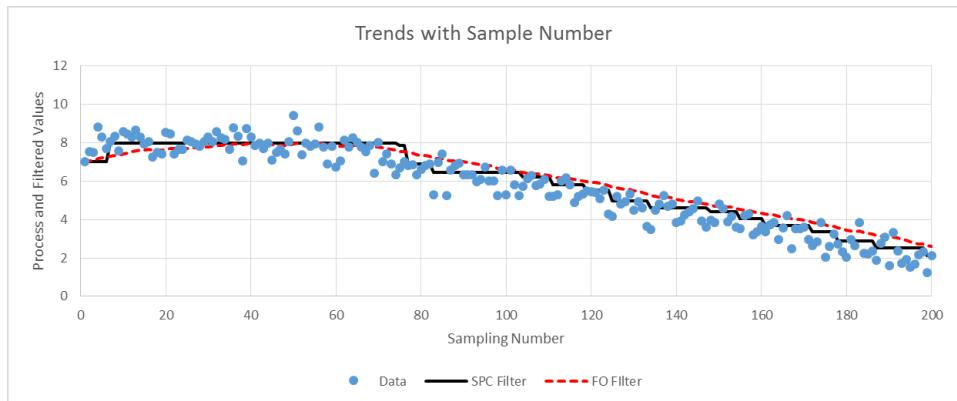
There are two more entries in the left hand column. The SPC filter uses an EWMA formula to update the variance. If you wish to give it the true value of the variance enter "Y" in the "known variance" cell. A "N" is more realistic. Finally, if you wish to impose a ramp or step change in the PV nominal value, place an "R" or and "S" in that cell.

Press the "Run Realizations" button to start the program. Each realization will be revealed in the graph. Here are two examples:

In the first, a step change happens at sample 100, when the PV value changes from 8 to 4 and its standard deviation goes from 1 to 0.5. The trigger value is 2. It is nominally initialized.



In the second, the initial PV value is also 8, but at sample 50 it begins a ramp to a value of 2. The sigma starts and stays at 0.5. It is nominally initialized.



Both figures reveal the character of the SPC filter. After a delay (needed to confidently determine that a change is needed) the SPC filter jumps to the new value. By contrast the FO filter shows the lag in a step response and the characteristic offset in following a ramp.

The table highlighted in yellow outputs the results. The upper row reports values for the SPC filter and the second row for the FO filter.

Table 3. Output data.

SPC F =	0.638957	0.652486	0.043875	0.264293	10.76
FOF =	0.882619	0.867426	0.04657	0.46528	48.92
	rms dev from true per run	rms dev from true, avg of all runs	Travel, average of all	abs deviation from true, average of all	Settling time average of all
Realization =	100				

The rms deviation from the true value is calculated as

$$rms = \sqrt{\frac{1}{199} \sum_{i=2}^{200} (x_{true} - x_{filtered})^2}$$

Only in a simulation can the true x-value be known. The simulation realization lasts 200 samples, but since the summation starts with the second, the averaging coefficient is 199, one less than 200.

The realization-to-realization results reflect the vagaries of small numbers, so the second column reports the average rms value over all realizations. The number in the lower left of the table displays the realization number.

Travel is the cumulative number of changes that the filtered value makes, also normalized for a realization

$$Travel = \frac{1}{199} \sum_{i=2}^{200} |x_{filter,i} - x_{filter,i-1}|$$

And averaged over all realizations. Where unnecessary changes in a system are a critical concern, "travel" indicates how much wandering the value did. If it moves from 5 to 6, travel is 1. If it moves from 5 to 7 to 6 (with the same initial and ending values) the travel is 3.

An alternate to the rms deviation is the absolute value of the deviation. The rms calculation squares the deviation, making a deviation of 2 four times as important as a deviation of 1. However, often, the value of the deviation is an indication of undesirability. The absolute value of the deviation from true is often termed the integral of the absolute error:

$$IAE = \frac{1}{199} \sum_{i=2}^{200} |x_{true} - x_{filter,i}|$$

The value presented is also averaged over all realizations.

Finally the right-most column reports settling time. This is the number of samples for the filtered value to get within a half-sigma of the true value after a step change. It is only relevant if there is a step change. A blank value is reported for ramp changes or if there is no step value.

If you wish to explore your own equivalence data (such as in Table 2) input a large number of realizations, "p" for perfect initialization, zero for the pause time (unless you want to take the time to observe each realization), and set x1=x2 and sigma1=sigma2. Enter the desired trigger value. Choose what assessment metric you wish to use (rms value, travel, IAE) and adjust the lambda value to make the average assessment statistic of the fof equivalent to the SPC filter. Because of the statistical vagaries, with 5,000 realizations, the third digit in the assessment statistics will be uncertain. Accordingly, I only report two significant digits for lambda values in Table 2.

N realizations =	5000	trigger =	2
Initialization (n/p) =	p	x1 =	8
pause time =	0	sigma1 =	1
known variance =	N	change =	100
ramp/step =	S	x2 =	8
		sigma2 =	1
		fof lambda =	0.053

The algorithm in VBA is:

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IF first_call THEN
    N = 0
    XOLD = 0.0
    XSPC = 0.0
    V = 0.0
    CUSUM = 0.0
    M=11
    FF2 = 1.0 / (M - 1) / 2.0
    FF1 = (M - 2) / (M - 1)
    First_call = "FALSE"
END IF
Obtain X
N = N + 1
V = FF1 * V + FF2 * (X - XOLD) ^ 2
XOLD = X
CUSUM = CUSUM + X - XSPC
IF ABS (CUSUM) > TRIGGER * SQR (V * N) THEN
    XSPC = XSPC + CUSUM / N
    N = 0
    CUSUM = 0.0
END IF

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