Filtering a Digital Signal in Control Systems

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CONTROL magazine, Develop Your Potential Series, Vol. 30, No. 12, December, 2017, pp 16-17.

First-Order Filter

Noise can be viewed as random perturbations on an otherwise constant signal. If there were no noise, the signal value would remain unchanged. However, noise makes the value fluctuate above and below the background, true value. Noise may be the result of mechanical vibrations, stray electromagnetic impulses, flow turbulence, or any number of causes. Further, the true value of the signal may be changing in time. The problem is that you cannot see the true signal, all you can know is the noisy measurement or calculated value. Figure 1 illustrates a slowly changing signal being confounded by noise. The true value of the signal is represented by the dashed line, illustrating that it is unknown. The control system can only know the fluctuating marker values, representing the measurement.

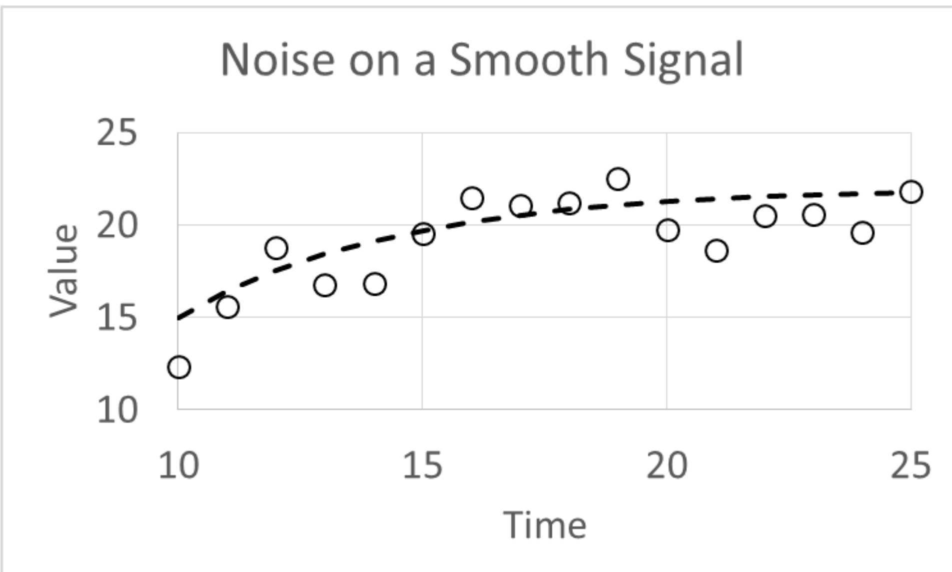

Figure 1 – A noisy response (markers) to an otherwise smooth signal (dashed curve)

The problem is that noise confounds action. For example, between times of 19 and 21, the measurements seem to indicate a significant drop in value, while it is actually rising slowly.

We would like to remove noise from signals, variables, and action; and averaging is a common way to temper the fluctuation.

$$\bar{X}_j = \frac{1}{N}\sum_{i=1}^{N} X_{j+1-i}$$

Here, the signal value is X, and the index j indicates the time. The index i is the counter for the 1st, 2nd, …, up to Nth past values that are to be averaged. Note that all N X-values need to be saved, and updated with each increment in time.

Averaging, however, was not possible with analog instruments of yester-year, and even in the digital age, represents a computational burden of storing many, N, variables. Filtering is a computationally simpler approach to "averaging", and can be developed a number of ways. Here is one: Take the most recent X-value out of the sum, and recognize that if the signal is not changing (at steady state) then the past average is a reasonable estimate for the sum.

$$\bar{X}_j = \frac{1}{N}\sum_{i=1}^{N} X_{j+1-i} = \frac{1}{N}X_j + \frac{1}{N}\sum_{i=2}^{N} X_{j+1-i} \cong \frac{1}{N}X_j + \frac{N-1}{N}\bar{X}_{j-1} = \left(\frac{1}{N}\right)X_j + \left(1-\frac{1}{N}\right)\bar{X}_{j-1}$$

Now all that has to be remembered is the past value of the average. The approximately equal sign reveals that even at steady state, the noise on the past average is different from the noise on the most recent, so the substitution of the average for the sum is not exact. Further, this substitution assumed that the signal is at steady state, which it may not be. Accordingly, don't call this approximation the average, call it a filtered value. I'll also rename the $1/N$ value as a filter factor, λ. The resulting equation is a first-order filter.

$$X_{f_j} = (\lambda)X_j + (1-\lambda)X_{f_{j-1}}$$

This equation is the digital representation of a resistor-capacitor circuit, or also a pneumatic tank-and-restriction device, used to temper noise. It is the numerical solution to the ordinary differential equation (ODE) of the RC circuit. It's mathematical form is also termed an autoregressive-moving-average of orders 1 and 1 – ARMA(1,1). The equation could be presented as its Laplace or z-transform version. Accordingly, there are many diverse representations for it, and you may find one or another classification names in the product bulletins of your devices. The variable λ, may be indicated as the filter factor, f. Alternately, the value and complement may be reversed so that the $(1-\lambda)$ term multiplies the most recent measurement. Still, the device might ask the user to choose the filter time-constant, from which it would calculate $\lambda = 1 - e^{-\Delta t/\tau}$. In any case, the user must specify the filter coefficient value; so, be sure that you understand which version your device is using.

The challenge for the user is to choose the right value for the filter coefficient. Filtering does not remove noise, it tempers it. So, it would seem that more filtering (a smaller λ value) is better to see the true signal. However, filtering induces a lag to X_f, masking the reality of an immediate change in the process. As filtering is increased to attenuate noise, the lag is also increased. The lag is undesired. It confounds the controller, and any other action that a person might take when observing a signal.

Figure 2 indicates the impact of three choices of filter coefficients on a signal that makes a step change to a steady value. The true, but unknowable, value is the dashed line, and the markers represent the data. Note that as filtering is increased, fluctuations on X_f are reduced, but the undesirable lag is increased.

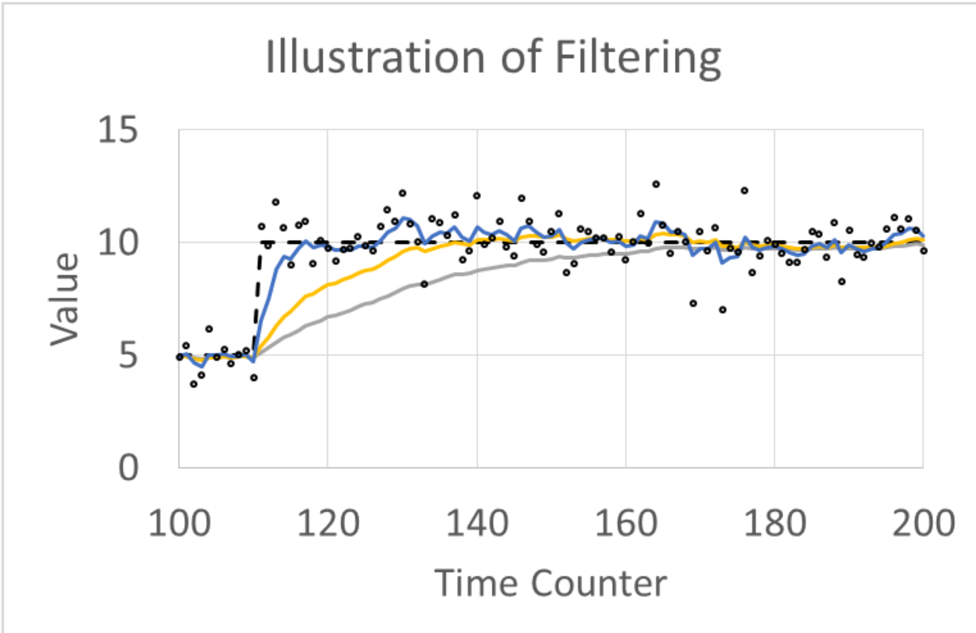

Figure 2 – Filtering and the Impact of Coefficient Values

Ideal equations for the time-constant for the filter lag (obtained by numerical representation of the ODE) and the variance reduction (obtained by propagation of variance on the filter equation, assuming steady state and un-correlated noise) are:

$$\tau = -\Delta t / ln(1-\lambda)$$

$$\sigma_{X_f} = \sigma_X \sqrt{\frac{\lambda}{2-\lambda}}$$

One could desire a particular variance reduction, then calculate the required λ-value from one equation, and the consequential lag from the other. But, idealizations make the equations approximations; and more realistically, the filter coefficient will be heuristically adjusted on-line to meet a visual and subjective desire for noise reduction.

Since process noise amplitude changes with operating conditions (pH, nonlinear signal processing, flow turbulence, and many other aspects), expect to readjust the filter coefficient value as changes in measurement signal would indicate. But also realize, significant changes in the consequential time-constant could affect controller tuning.

Filtering may be effected on the process sensor, data transmission, or controller. Be aware of choices that other technicians and operators may make. Tune the controller after the filter coefficient is set.

In filtering, the user needs to understand: 1) the balance of desirables (reduced fluctuation) and undesirables (increased lag); 2) the equation that the device is using; and 3) the impact that the lag may have on controller tuning and performance, and on the interpretation or the process by human observers.

Median Filter

Here the objective is not to find the average of a noisy signal but eliminate the occasional outlier or spurious signal of an event due to a missed data fault, electromagnetic pulse, or other such abnormal one-time, irregular event.

The median filter reports the middle of the most recent values – not the middle in chronological order, but the middle in value. For instance, if the most recent three values are 5,6,3 the middle value 5 is reported regardless of its place in the sequence. Often, redundant sensors are used in which the middle of three measurements is taken as the process value, in a procedure termed voting. However, voting is a special case of parallel measurements at the same time. In a median filter, the middle-of-three is from a sequence of data. A median filter could be based on 3, 5, 7, or so sequential data. If you suspect that two outliers could happen sequentially, because of the persistence of some common cause, then a median of 5 will reject them. The median filter does temper noise a bit, but the application intent should be to remove outliers.

Figure 3 illustrates the median filter (middle of 3) applied to the same set of data. Note: 1) When the signal makes a step change at the 30th sampling, the filter has a delay of about half the number of data. 2) The outlier at sample 120 is wholly ignored. 3) Throughout, the vagaries of the signal substantially mimic the measurement.

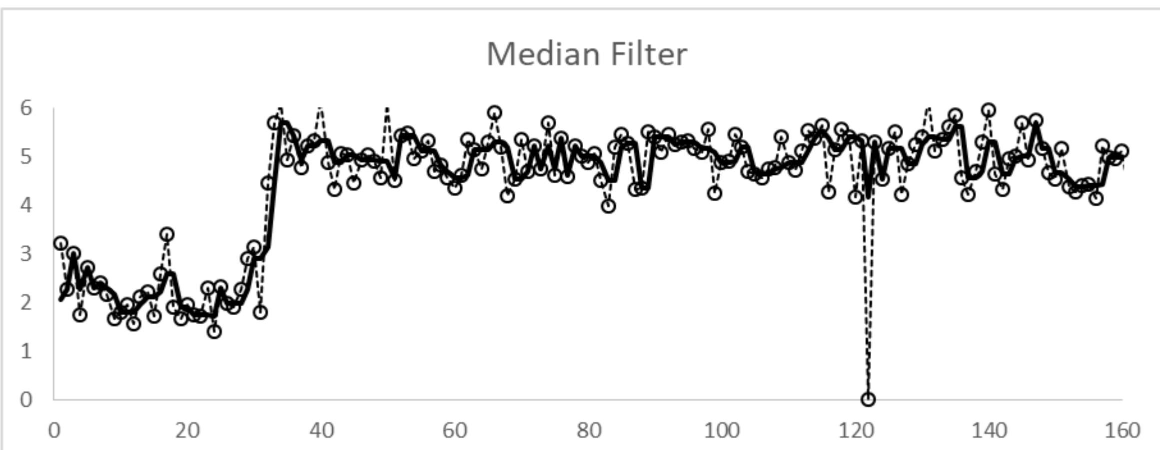

Figure 4: Characteristic performance of a median (middle of 3) to a process step change and spike.

Note: The median filter removes outliers, and rapidly tracks real changes. However, the user must choose an N that is large enough to exclude persistent outliers. Masking outliers can misrepresent important features, and noise is not removed.

For more on data filtering in the process industry, see Alford, J. A., B. N. Hrankowsky, and R. R. Rhinehart, "Data Filtering in Process Automation Systems" InTECH (The ISA periodical) July-August, 2018, Vol. 65, Issue No. 4, pp. 14-19.