Bayes Belief

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Introduction

Belief is the confidence that you have in making a statement of fact about a supposition. Here are some examples of statements that we might make:

> "These symptoms are just seasonal allergies. I am not contagious."
> "The average benefit of Treatment Y is larger than that of Treatment X."
> "The process has reached steady state."

Each statement is either true or false. They are seasonal allergies or not. Y is larger than X, or it is not (not-larger includes equivalent). It is at steady state or it is not (nearly at steady state is still not).

And for each we might claim to be very certain of the supposition (perhaps 99% sure), or somewhat certain (perhaps 80% sure), or even not sure whether it is or is not (perhaps 50% sure).

Bayes Belief, B, is that confidence scaled by 100%, so the Belief value is $0 \leq B \leq 1$. If you tend to question a statement, the belief that it is true, B, might be 0.25. If you are fairly certain, B might be 0.97. If you are not so sure about something, and it could be a 50/50 call, then $B \cong 0.5$.

Because we take action on suppositions, when the consequences are important, we want to be fairly certain that the statement about what we suppose represents the truth about the reality. When you are not certain, you perform tests, take samples, get other's opinions, etc. to strengthen or to reject your belief in the supposition. But tests are not perfect. There is always some uncertainty about the results.

Examples of Test Uncertainty

For example, consider a long multiplication test for 5th graders, such as

$$\begin{array}{r} 426 \\ \underline{x75} \end{array}$$

If a kid understands the procedure and can do it, there is still a chance that they will accidentally shift the two lines to be summed, or forget to carry, or something. They may make a mistake unrelated to the concept and procedure of long multiplication, perhaps in 2 out of 10 examples.

Even if a kid knows the procedure, on a multiplication test, they may get false negatives (a wrong answer on a particular problem) in 20% of the instances. Alternately, someone who is confused about the procedure might guess at the method, and get the right answer once. Perhaps in 1 out of 10 problems. This means false positives 10% of the time.

Accordingly, a one-problem test cannot be used to determine if the student has learned long multiplication, or not. A student who knows it, might make an error and get it wrong. A student who does not know it, might be lucky and get a right answer. So, looking at the results of just one test problem, you cannot be sure of the student's ability. But with 10 test problems, it is very unlikely that a confused student will get more than 5 right, or that a student who knows it will get more than 5 wrong. If they get 8 or 9 right, or get 8 or 9 wrong, the teacher can be fairly certain of the case, and the teacher can then make rational decisions related to semester grades and to future lessons.

As another example, the manufacturer of a particular procedure for detecting the presence of colorectal cancer reports that the test correctly detects the disease in 92% of the patients with cancer, and gives a correct negative result in 87% of the patients without the disease. (Exact Sciences Laboratories, Cologuard Patient Guide, 2019). The 92% correct positives means 8% false negatives. (On 8% of patients with the disease, the test will indicate they do not have it.) Similarly, the 87% correct negatives means 13% false positives. (On 13% of patients without the disease, the test will indicate they do have it.)

Here is a matrix of the probabilities of the test giving true and false indications.

Table 1. Probabilities of correct identifications and false positives and negatives for a medical test

Test / Truth	Test is positive (claims to detect cancer)	Test is negative (does not detect cancer)
Patient has cancer	0.92	(1-0.92) = 0.08
Patient does not have cancer	(1-0.87) = 0.13	0.87

Here is a third (and final) example of uncertainty in test results: A test for steady state (SS) might look at the past several data points. At SS the time-rate of change, data slope, ideally is zero, $S = 0$. But, because of noise on the data, the slope will not be exactly zero; so, you might accept SS if the test results are $-0.1 \leq S \leq +0.1$. So, if the measurements indicate $S = -0.03$ you say that is just noise, and the test indicates SS. But, at SS, a particular confluence of perturbations in the data might indicate the local slope is $S = 0.15$, and the test would reject the true condition of SS. Maybe, given a true SS, the test will indicate SS 85% of the time, and reject SS 15% of the time.

On the other hand, if the process is in a transient state (TS), the slope will be much greater than a SS value, the slope will be beyond the $-0.1 \leq S \leq +0.1$ limits, and the test result will claim TS. However, even in a transient when the process variable is moderately changing, the noise pattern on the past few samples might have a counter trend, and the rate of change might incorrectly indicate SS. Maybe, given a true TS, the test will indicate TS 95% of the time, and SS 5%.

Here is a matrix of the probabilities of the test giving true and false indications.

Table 2. Probabilities of correct identifications and false positives and negatives for a process test

Truth \ Test Result	Test Claims SS	Test Claims TS
Process Is SS	0.85	(1-0.85) = 0.15
Process Is TS	(1-0.95) = 0.05	0.95

The key message in this section is that tests are not perfect. There is uncertainty in test results.

Concept of Test Results Affecting Belief

If you somewhat believe something is possibly true, perhaps $B = .75$, and you do a test which supports your belief (which indicates it is probably true), then your belief value rises, perhaps it becomes $B = 0.99$, which may be strong enough belief to take action on it. But, if the test indicates it is probably not true, your belief falls. Maybe it falls to 20%, somewhere in-between zero and your former belief of 75%. If a second test also indicates the hypothesized benefit is probably not true, your belief falls again, perhaps to 1% which may be enough to justify rejecting the initial supposition.

Note: The belief should never go to 100% or to 0%. Either extreme would mean that the test was infallible.

Four Separate Concepts

There are two different aspects to knowledge in this analysis. Keep them separate. First, there is the truth about the situation. The patient either has the disease or does not. The process is either at SS or not. Treatment Y is either better than Treatment X, or not. The truth is stated in the left-most column of the tables. But the truth is not known. So, we use a test.

The second knowledge aspect is the test result. The test either indicates positive or negative results, such as Y is better than X or it is not. The test outcome is stated in the top row of the tables. But the test is not infallible. The test results may, or may not, represent the truth.

Also, there are two different aspects to probabilities in this analysis. Keep them separate. The first is the entries in the table, the probability of a test giving a correct or wrong outcome. You get these values from extensive testing when the truth is known. For example, false negatives and positives for the colorectal cancer test were determined by comparing the noninvasive test to the results of colonoscopy on many people with and without the disease.

The second probability aspect is the belief, the personal confidence that something is true. But just because the person wants to believe it is true (or not true) does not make it so. (Humans used to believe that the sun revolved around the Earth, but the belief did not make it so.)

Method

Belief changes with successive test results. The Bayes Belief method for updating belief after test results is.

$$B_{after} = \frac{B_{prior} \cdot p(the\ test\ affirms\ supposition\ if\ supposition\ is\ true)}{B_{prior} \cdot p(\ test\ affirms\ ) + (1 - B_{prior}) \cdot p(test\ affirms\ supp\ if\ supp\ is\ false)} \tag{1}$$

Where B_{prior} is the belief prior to the test results, and B_{after} is the belief after the results. The numerator is the expected outcome if the supposition is true. The denominator is the sum of all such test outcomes that are possible (whether the supposition is true or not true).

(The method was introduced by Thomas Bayes (1702-1761) an English statistician and Presbyterian minister.)

The belief could be in either of the two mutually exclusive cases. For example, that Y is better than X, or that X is better than Y. Or, that the patient has cancer, or does not have cancer. Or that the student correctly has learned the long multiplication procedure, or has not. It does not matter which supposition you choose.

The test results could either support or counter the belief. The term $p(the\ test\ affirms\ supposition\ if\ supposition\ is\ true)$, and the shorter version of the same probability, $p(\ test\ affirms\ )$, is the probability from the table that the test results will affirm the supposition if the supposition is the truth. For example, if the supposition is that you have cancer, and the test result is positive, the probability from Table 1 is 0.92. If the supposition is that you are cancer free and the test indicates you do not have it, the probability from Table 1 is 0.87.

Alternately, the term $p(test\ affirms\ supposition\ if\ supposition\ is\ false)$ is the probability from the table that the test result is the same even if the the other supposition is the truth. For example, if the supposition is that you have cancer, and the test result is negative, the probability from Table 1 is 0.08. If you believe you are cancer free and the test indicates you do have it, the probability from Table 1 is 0.13.

Example 1: If the supposition is that the process is at steady state (SS), and the belief in that claim is 75%, and then a test affirms the supposition, what is the new belief value after the corroboration?

From Table 2.

$$p(test\ affirms\ the\ supposition\ if\ true) = 0.85, \text{ and}$$
$$p(test\ affirms\ supp\ if\ supp\ is\ false) = 0.05$$

So, if the supposition is "at SS" with $B_{prior} = 0.75$, then Equation (1) directs that

$$B_{after} = \frac{(0.75)(0.85)}{(0.75)(0.85) + (1 - 0.75)(0.05)} = 0.980$$

The result is rounded. And, if a second test affirms "at SS" then Equation (1) reveals

$$B_{after} = \frac{(0.980)(0.85)}{(0.980)(0.85) + (1 - 0.980)(0.05)} = 0.999$$

That would be a very strong belief.

By contrast, if the initial belief that the process is at SS, with $B = 0.75$, and a test rejects that supposition, then Equation (1) directs

$$B_{after} = \frac{(0.75)(0.15)}{(0.75)(0.15) + (1 - 0.75)(0.95)} = 0.321$$

The belief drops. A 0.321 value means little certainty that the belief is either correct or incorrect. The decision would be to observe more data before taking action.

If a second test also rejects SS, then the belief becomes

$$B_{after} = \frac{(0.321)(0.15)}{(0.321)(0.15) + (1 - 0.321)(0.95)} = 0.070$$

And then a third, $B_{after} = 0.012$, which may be low enough to reject the initial supposition, and accept that the process is probably in a transient state.

Example 2: If the supposition is that the person is healthy (does not have colorectal cancer), and the belief in that claim is 50%, and then a test affirms the supposition, what is the new belief value after the corroboration?

From Table 1.

$p(test\ affirms\ the\ healthy\ supposition) = 0.87$, and
$p(test\ affirms\ supp\ if\ supp\ is\ false) = 0.08$

So, if the supposition is "healthy" with $B_{prior} = 0.5$, then Equation (1) directs that

$$B_{after} = \frac{(0.5)(0.87)}{(0.5)(0.87) + (1 - 0.5)(0.08)} = 0.916$$

The result is rounded. That would be a comforting belief.

And, if a second independent test also affirms "healthy" then Equation (1) reveals

$$B_{after} = \frac{(0.916)(0.87)}{(0.916)(0.87) + (1 - 0.916)(0.08)} = 0.992$$

That would be a very strong belief.

By contrast, if the initial belief that the person is "healthy", with $B = 0.5$, and a test rejects that supposition, then Equation (1) directs

$$B_{after} = \frac{(0.5)(0.13)}{(0.5)(0.13) + (1 - 0.5)(0.92)} = 0.124$$

The belief drops. A 0.124 value means little certainty that the healthy supposition is correct, but a somewhat certainty that the supposition is incorrect. The decision could be to repeat the test to get corroborating data before taking action. But, more likely the decision would be to perform a more invasive colonoscopy test for even greater certainty.

However, if a second independent test (with the same test probability values) also rejects "healthy", then the belief becomes

$$B_{after} = \frac{(0.124)(0.13)}{(0.124)(0.13) + (1 - 0.124)(0.92)} = 0.020$$

Which may be low enough to reject the initial supposition, and accept that the person has the disease.

In general, the table of probabilities of false and correct test outcomes is indicated by Table 3.

Table 3. Generic probabilities of correct identifications and false positives and negatives

Truth \ Test Result	Test claims it is true	Test claims it is false
Something really is True	$p =$ p(true claim, if true)	$(1-p) =$ p(false claim, if true)
It really is False (not True)	$(1-q) =$ p(true claim, if false)	$q =$ p(false claim, if false)

Note: Sequential tests must be independent. What led to a test outcome on prior tests cannot also be influencing subsequent tests. For example, if the environment is distracting the 5th grader who has "got" the procedure and who is taking the multiplication test, diverting concentration on each problem, then there may be many mistakes and the test outcome a false negative. As another example, in the SS-TS test, if there is autocorrelation in the process data, then what influenced the prior test will persist and have the same influence on a subsequent test. And a third: A repeated analysis of one sample may return the same value, but the sample may have been contaminated and not represent the process. Get another independent sample.

Choosing Thresholds on Belief for Action

What is an adequate confidence level to be able to take action? If $B = 0.99$ then you are fairly certain that the supposition is true. Should the triggers to accept a supposition be $B = 0.8$ (moderately sure) or $B = 0.9999$ (very, very sure), and to reject the supposition be $B = 0.2$ or $B = 0.0001$? I will offer that conservative generic values seem to be $B = 0.999$ or $B = 0.001$. Note: These are complementary, representing the same extremes, $(1 - 0.001) = 0.999$.

But alternate belief levels may be appropriate to your application. How can an appropriate threshold on belief be determined?

To take action, be sure that the consequences of a decision, tempered by the probability of a decision, are acceptable. I'll offer that there are three aspects to consider. First is the number of trials. The more extreme are the belief thresholds, the greater will be the number of trials to get that belief value. Trials cost in effort and material, and cost in time delaying a decision. Second is risk. Risk is the probability of an undesired choice (you can substitute Belief for the probability) times the consequences of that undesirable event. If you do not have extreme values for the belief threshold, then there is more of a chance that you could accept the wrong action. Third is benefit. Benefit is the probability of a desired choice (you can substitute Belief for the probability) times the consequences of the desirable outcome.

There are four actions: Two are correct - Accepting X when it should be X, or Y when it should be Y. And two are wrong - accepting X when it should be Y, or Y when it should be X. But for each there are two ways to come to the decision, because the belief leading to each of those choices could be a high extreme or a low extreme. For example, if you believe that X is true (and it is), and if the belief that X is true exceeds the upper threshold, then you accept X when it should be X. However, if you believe that Y is true (and it is not), and if the belief that Y is true exceeds the lower threshold, then you accept X when it should be X. In either case, whether the initial belief is correct or wrong, the decision is correct.

Values of the thresholds have opposing effects. The more extreme, the lower the chance of a wrong decision and the better the chance of a right decision, but the greater will be the number of the trials (hence the cost of the tests). I suggest setting the thresholds of Belief to take action by the consequences of taking a right or wrong action (accepting X when it should be X, or Y when it should be Y; rejecting X when it should be X, or Y when it should be Y) and the cost of trials to meet a threshold belief. Choose the threshold to maximize the benefit of a right decision and minimize both the penalty for a wrong decision and the cost of trials.

This concept is stated as an optimization, a minimization (minimize the undesirables and the negative of the desirables):

$$\underset{\{Belief\ threshold\}}{min}\ J = cost\ of\ trials + risk - benefit \tag{2}$$

The cost of the trials depends on the number of trials. In my simulation investigations, I find that the number of trials is well correlated to the test probabilities of correct results (p and q from Table 3). The model for the average number of trials to hit a belief threshold from an initial belief of 0.50 is

$$\bar{n} = ae^{b/(pq)} \tag{3}$$

With coefficients a and b dependent on the threshold value, T, chosen. Also, empirically:

$$a = 0.05953 - 0.045414 * ln(T) \tag{4}$$
$$b = 1.9553 - 1.9328 * T \tag{5}$$

So, if p and q values are 0.92 and 0.87 (from Table 1) and the belief threshold is $T = 0.001$ (and the other is the complementary 0.999), $a = 0.3493223$, $b = 1.940366$, and $\bar{n} = 4.28427 \ldots \approx 4$ trials. On average, after a bit over 4 trials the test results will combine to a 99.9% belief.

There is some random variation in my data, and the replicate variation on $\bar{n}$ seems to be less than 1 trial. Further, I have not investigated the impact of the initial Belief on this model. But experience indicates that if the initial belief is 0.5 then the outcome of one trial generally moves it to what might be an alternate initial belief of fairly high confidence. All in all, $\bar{n}$ might be one higher, or one less than Equations (3-5) predict.

There is a “cost” to running the trials. It includes true costs associated with material and labor, but also auxiliary considerations and impacts (such as time to a decision, diversion of the production process for testing, lab analysis, anxiety to present a confident decision prior to annual appraisal, etc.). It may be difficult to quantify such auxiliary concerns in monetary terms to be equivalent to direct cost of trials. Further, in Equation (2) the cost of the trials must be dimensionally consistent with the quantification evaluation of risk and benefit. I would recommend the Equal Concern approach to normalizing terms in Equation (3), to making them have consistent units and weighting. Instead of using monetary values, use the number of trials that causes equivalent concern or benefit.

$$\underset{\{T\}}{min} \; J = n\ of\ trials + T(equivalent\ n\ trials\ if\ a\ wrong\ action) \\ -(1-T)(equivalent\ n\ trials\ if\ a\ right\ action) \quad (6)$$

Here is an example of normalizing penalties and benefits to match the number of trials: The organization is running trials to generate data to support matching a steady-state (SS) model to the process. The experiment starts by setting process inputs, observes the process as it begins its transition (a TS) from the prior state to the new state, then waits until the process settles to a SS. At SS you collect data that fits the SS purpose, then implement the next set of experimental conditions. The supposition is that the process is at SS, but you start each trial with a belief that it is a moderately low value, perhaps $B_{SS} = 0.25$, because you saw evidence of the TS caused by new inputs, and suspect the process should not be at SS. When B_{SS} rises to some threshold high value, perhaps $B_{SS} > (1-T) = 0.999$, then you take action again – record data and implement next trial conditions. The supposition is that you are at SS.

Table 4 represents the truth and the decisions. Note the 4-cell similarity and Truth as the left most column similarities to the prior tables. However, the upper row in Table 4 is not the test outcome, but the decision.

Table 4 – Decision Analysis for the SS Example

	Decision	
Truth / Actual	Accept SS Initial Belief, or Reject TS Initial Belief (which means Accept SS) B(SS)=(1-T), or B(TS)=T.	Accept TS Initial Belief, or Reject SS Initial Belief (which means Accept TS) B(SS)=T, or B(TS)=(1-T).

At SS	Good. You collect appropriate data, and begin next test. The joy is equivalent to another test, benefit=1. P(of being here)=(1-T).	Bad. You extend the time and cost of the test, and delay getting data. The Concern, the consternation and defense of delaying the program unnecessarily is equivalent to 50 tests. P(of being here)=T.
In a TS (Not at SS)	Bad. You collect data under TS conditions, which does not reflect SS. The models will be corrupted with data representing a transient. We strongly desire the model to be correct. This has a concern that is equivalent to 100 tests. P(of being here)=T.	Good. You continue to wait, appropriately, until SS. Although the cost of tests increases, it is appropriate for the conditions. The joy is effectively zero. P(of being here)=(1-T).

With the concerns as stated in Table 4, the optimization statement is

$$\min_{\{T\}} J = ae^{b/(pq)} + T(1+0) - (1-T)(50+100) \quad (7)$$

S.T: $a = 0.05953 - 0.045414 * ln(T)$

$b = 1.9553 - 1.9328 * T$

With p and q values from Table 2, and an initial belief of 50% the optimal threshold is $T = 0.003639$, which indicates on average 3 to 4 sequential, independent tests will be needed.

Consider another example – the colorectal cancer test. The p and q values are from Table 1. Table 5 indicates possible equivalent impact of the tests to the right and wrong decisions.

Table 5 – Decision Analysis for the Medical Test Example

	Decision	
Truth / Actual	Accept Healthy Belief, or Reject Have Cancer Belief	Accept Have Cancer Belief, or Reject Healthy Belief
Cancer Free, Healthy	Good. The joy is equivalent to zero more tests, benefit=0. P(of being here)=(1-T).	Bad. You think you have cancer when you don't, the anguish and additional invasive testing is equivalent to 30 more tests. P(of being here)=T.
Have Cancer	Bad. You have cancer, but think you do not. As you live unaware, the cancer is growing. Eventually it will	Good. Although the outcome is undesirable, at least you are properly diagnosed and can start treatments.

	be detected, but it is much better to start treatments as soon as possible. This has a concern that is equivalent to 50 tests. P(of being here)=T.	The joy is effectively zero, but the benefit of knowing is worth 10 more tests. P(of being here)=(1-T).

Now the optimization statement is

$$\underset{\{T\}}{min} \; J = ae^{b/(pq)} + T(0+10) - (1-T)(50+30) \quad (8)$$
$$\text{S.T:} \quad a = -0.0031724 - .051029 * ln(T)$$
$$b = 1.9426 - 2.2340 * T$$

With an initial belief of 50% and the benefit and penalty values from Table 5, the optimal threshold is $T = 0.00612$, which indicates that on average just over 3 independent tests should be done.

Of course, the medical profession would take an alternate action if one test contradicted the supposition. If the person is presumed healthy then the initial belief might be 0.9 and one test that affirms that raises the belief to 99%. Or, if one test rejects the supposition, that lowers the healthy belief to 56% which likely would result in a standard colonoscopy rather than a repeat stool test. Alternately, if Table 5 values are 0, 1, 2, and 2 for the test equivalents (replacing the 0, 10, 50, and 30 values because of an alternate viewpoint on consequences), then the optimal threshold is $T = 0.1537$, which indicates that on average $1.13 \cong 1$ test is appropriate. Perhaps this is about how the medical system has normalized the cost of tests with the penalty and benefit of right and wrong outcomes given that there are backup tests and a justification for higher initial belief values.

An alternate procedure

The binomial distribution can be used to determine the probability of s number of successes out of n number of trials, where the probability of a success is p. If the supposition (hypothesis) is that X is true, then the probability of s or fewer number of affirmations in n trials is

$$CDF = \sum_{x=0}^{s} \binom{x}{n} p^{x} (1-p)^{(n-x)}$$

If, after n trials, this quantity is less than 0.001, then reject the hypothesis.

You need to look at both possibilities. If the supposition is that the process is at SS then from Table 2, the probability that the test gets it right, the probability of a test indicating SS is $p = 0.85$. Alternately, if the supposition is that the process is in a TS then from Table 2, the probability that the test gets it right, the probability of a test indicating TS is $p = 0.95$. If, for instance, the supposition is that the process is at SS, and only 2 out of 7 tests support that, the probability of 2 or fewer successes out of 7 trials, if p=0.85 is 0.0012. Reject the supposition. Similarly, if the supposition is that the process is in a TS, and only 2 out of 5 tests support that, the probability of 2 or fewer successes out of 5 trials, if p=0.95 is 0.0011. Reject the supposition.

In my simulations, this is equivalent to the cumulative Bayes Belief. With the same threshold of 0.001, the Bayes method seems to take about one more test to cross the threshold, but it also has a false conclusion of about half of the threshold.

Perspectives on Bayes Belief

Often the probabilities in the four categories in Table 3 can be determined from controlled testing on known situations. Only two probabilities are needed. The others are the complements. But as often, they can be reasonably estimated from experience. In either case do not think that the probabilities are perfectly known. They have error. There will be uncertainty on the probabilities. Even if they represent experimental data from controlled testing, the test will have limited size, and samples may not have included all population attributes. But, in my experience the uncertainty on the reasonable values does not undermine this method for propagation of Belief. Alternate p-values might lead to needing one more or one fewer number of tests to provide adequate confidence to take action.

After each trial, you will know more about your process and the probabilities. So, perhaps you can have information to update probabilities using your developing knowledge.

If the test has a large probability of false outcomes (negatives or positives) or if there is truly no difference in the treatments, then the calculated belief could as easily go toward one extreme as the other. However, after enough tests Belief will eventually hit one extreme B-value or the other, even if there was no difference. If there is a definite difference and the test has reasonable probability of correct results, then the progression of Belief should consistently move toward one extreme and get there within at most about a handful of test results. A sequence of beliefs that keep reversing direction is not what is expected. So, if the sequence of test outcomes does not seem to meet what you would expect if the supposition was definitive, then accept that it is indefinite. Don't continue testing for a long time, until a random walk leads to a misleading extreme B-value.

Many accept the Bayes Belief approach as a very good guide to updating belief with sequential results, and then using the belief to make decisions. Alternately, although purists seem to accept the mathematical model, many experts object to the method because of the uncertainty on the values for either probabilities, or the initial belief, and the user-values that select the B threshold to take action.

The appropriate threshold value is strongly dependent on the test-equivalent choices for the consequences of right and wrong test outcomes.

Of course, humans might not want to follow this kind of logical rule. Often, when they know something is true, they consider themselves to be absolutely sure, and rather than admit they were wrong, they reject any data that would counter their personal belief. Or when they want something to be true, they reject any opposing data. Here are some examples I've encountered:

"Win or lose, our sports fans are kind and gracious, but our opponent's fans are always disrespectful poor sports."

"My kid is the best looking, smartest, and most athletic in the entire class."

"Reel mowers are better than rotary mowers."

If your boss, or significant other knows the truth, or wants a particular outcome, and the resulting action from an erroneous belief has less adverse consequences than the personal cost and effort of proving that person wrong, it might be best to let it go their way. Only martyrs let logic lead to a confrontation against authority. On the other hand, I would hope that each of us protect ourselves, our organizations, and society from action based on erroneous beliefs.

This is an approach to use fallible data to logically temper or support a fallible human belief. I believe it is a valid, rational, and valuable approach. I believe it is practicable, and that it can help eliminate irrational actions that are predicated on beliefs grounded in prejudice, technical folklore, misunderstanding, etc. Practice applying this propagation of belief in your personal and professional life. Understand the concept, then you will be able to qualitatively (subjectively, intuitively) apply it. You do not need perfect probabilities in the table. Develop your potential, then promote this approach to others in charge.

Suggested Exercises

1. The reader might want to explore Examples I or 2 to see the impact of alternate suppositions, alternate initial belief values, and alternate test probabilities.

2. From your personal experience choose values for a Table 1 or 2 associated with a 5th grader's test of long multiplication ability? How many test problems would be required to see if the student has acquired the skill or needs more instruction? Again, from your experience, create an equivalent to Table 4 or 5 for the 5th grader's long multiplication test. Consider the "costs" associated with the tests (paper and pencil or electricity consumption, student and grader time and bother, test creator time), and determine equivalent benefits and penalties for ability or lack thereof (think of the impact of additional testing on the unfortunate classmates who don't need additional monotonous, tedious classwork).

3. If there is a series of tests, the binomial distribution predicts the probability of a number of successes out of n trials. You might want to compare the number of trials that a Bayes Belief calculation method requires for a given belief threshold to that of the binomial distribution. In my exploration the results are similar when Belief starts at 0.5.

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Russ Rhinehart started his career in the process industry. After 13 years and rising to engineering supervision, he transferred to a 31-year academic career. Now "retired", he enjoys coaching professionals through books, articles, short courses, and postings on his web site www.r3eda.com.