Things We Do That Can't Be Done - 1
Science/Math Illegalities in Engineering

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Engineering ranks efficiency and practicality in calculations over scientific or mathematical perfection. However, our sensibleness with terms and equations can often lead others astray, and those more grounded in perfection might call us engineers downright sloppy! You need to be aware of such engineering conveniences when doing calculations.

This first part of this article discusses the terms and calculations, and the second part will discuss conveniences used in probability calculations for reliability and safety systems.

Terminology

A pound mass is not the same as a pound force. If you have one pound mass on Earth and place it on a scale, it weighs 1 lb. This means the scale springs have to push up with a 1 lb force to hold it. But on the Moon, the same mass would only require one-sixth of the force to hold it. It would weigh about 0.17 lb_f. Dual use of the term lb is inconsequential on Earth, but the meaning of mass and force is quite different. When using the British (Imperial) system of units be sure to explicitly state lb_m or lb_f. With the SI system of units, mass and force have separate names (for example kilogram, kg, and Newton, N).

There is another use of the term lb: We called our low-pressure steam line the 15-lb line because the pressure was 15 psig. I asked a new engineer to design a heater using the 15-lb line to dry solvent off filter fabrics. He interpreted the term to mean that the line had 15 lb_m in it. He calculated that there was only enough steam to dry about 10 of the filters, then there would be 0 pounds left in the line, and we would not be able to dry any more. Industrial terminology may be misleading to a novice.

Omitted Terms/Coefficients:

We use $F = ma$ and omit the dimensional unifier g_c which has the value of 1 (kg-m/N-s^2) in SI units. In the British system however, g_c=32.174 (lbm-ft)/(lbf-s^2). A one lb mass on earth has a weight of 1 lb force. But, without g_c, one of our fundamental equations, $F = ma$, would indicate the 1 lb_m mass has about 32.2 lb_f weight on Earth. The equation should be $F = ma/g_c$.

We take the logarithm of values, such as log(x), but the argument of the log function must be dimensionless. For convenience we omit the unity scalar log(x/1) because this is log(x)-log(1) and since log(1)=0, the log(x) has the same value as log(x/1). For example, pH, a measure of net acidity, is commonly accepted as the negative log of the hydrogen ion concentration, $pH =$

$-\log_{10}([H^+])$. But since the units on $[H^+]$ are moles per liter, this cannot be done. The truth is that the argument of the logarithm is the ratio of the $[H^+]$ of the solution to the $[H^+]$ at a reference concentration of 1 mole per liter. $pH = -\log_{10}([H^+]/[H^+]_{reference}) = -\log_{10}([H^+]/1)$, which for convenience reduces to $pH = -\log_{10}([H^+])$.

In the valve capacity equation we use $F = Cv\, f(x) \sqrt{\Delta P_v / G}$, where $f(x)$ is the valve characteristic, and x is the valve stem position. G is the fluid specific gravity, which is dimensionless. Since Cv has the units of F, perhaps gpm, the argument of the square root function needs to be dimensionless. But ΔP_v may have the units psi. The 1 psi scalar, $\xi = 1\, psi$ is omitted for convenience because it does not change the value. $F = Cv\, f(x) \sqrt{\frac{\Delta P}{G\xi}}$. But for mathematical consistency ξ needs to be there.

Commonly, g_c is also omitted in the friction factor relation for pressure losses $\Delta P = f \frac{L}{D} \frac{1}{2} \rho v^2$. But as it is, the equation does not convert the units of kinetic energy to that of pressure drop. It needs to be scaled by g_c.

The argument of an exponent must be dimensionless. It is important to use the value for the gas law constant, R, that is consistent with the units on the other variables in equations of state, vapor-liquid equilibrium relations, and reaction kinetic equations.

Deviation Variables:

psig is a deviation from atmospheric pressure, and °F is a deviation from about 458.67 Rankine. If your calculation requires true pressure and temperature, such as in the ideal gas equation of state, be sure to convert deviation measures to absolute.

Laplace transform notations are based on deviation variables. Consider step-testing a process to get first-order plus deadtime (FOPDT) models. The step might be initiated at 2:27 pm but that begins t=0, a deviation from 2:27. And the initial steady value of the process may have been .012 mole fraction impurity, with a 36% controller output, but the initial deviation values are considered to be zero. When converting Laplace notation to real variable calculatons, one must first subtract the reference value from the inputs, and after the calculations in deviation variables, add the reference value to the outputs.

Chemical reaction and thermodynamic equilibrium models require absolute temperature, unless the equation already contains the reference temperature and permits the use of degrees F or C.

Dimensionless/Unitless:

Unitless means that in a ratio of values the unit labels cancel. For instance, π is the length ratio of circumference to diameter, grade of a road is the length ratio of elevation change to distance, and Reynolds Number, $Re = \frac{du\rho}{\mu}$, is the rate of momentum ratio conveyed by the flowing fluid along the flow direction to that diffusing perpendicular to the flow direction. These unitless ratios are often termed dimensionless values because the units on the numerator and denominator cancel. But to use the ratios one must preserve their dimensional meaning.

Consider: Any measurement of quantity refers to that quantity. Canadian $ have a different value than US $ even though the label $ is the same. If the value ratio is 0.8 [$/$], it appears to be unitless, but to show how to use it the 0.8 must still carry the associated units of the value of a Canadian $ to the value of a US $. The value ratio is not unitless. It is 0.8 [$US/$CA]. But for convenience we often eliminate the labels and units in the act of converting. Search, "How do you convert psi to kPa?" The answer is, "Multiply by 6.89476". For convenience we don't show the dimensional units, but the answer is multiply the psi value by 6.89476 [kPa/psi].

A count is not dimensionless either. A count of 3 apples is not the same as a count of 3 oranges. Oranges are not the same as apples. In making a fruit basket of equal number of apples and oranges, the ratio is 1:1 but this is not a dimensionless 1. It is not 1 [pencil per apple] it is 1 [orange per apple]. The units on a ratio number must also reflect the items in the ratio. If you wanted to double the recipe, you'd multiply the number of oranges by 2 which has the units of [large/standard] to get 6 oranges. If you had 7 oranges and wanted to get the number of apples, you multiply the number of oranges by the number of apples per orange, 1 with the units [apple/orange]. If the multiplier 1 did not have units, then the multiplication 1*7 [oranges]=7 oranges, when it should be 7 apples.

Similarly, proportions and probabilities (based on count, cost, etc.) are not dimensionless. A probability is the count of number of events per number of trials. The expected count of outcomes with event A is the probability of A times the total number of trials. To convert total number of trials to expected count of A outcomes requires probability to have the units [count of A per total number of trials].

The units on a composition ratio may have the same measurement quantity, but the units are not dimensionless. 1 [lb of A] per 100 [lbs of B] is 0.01 [lbs A per lbs B]. If the ratio is 0.01 [dimensionless] then multiplying 1000 [lbs of B] by 0.01 would return 10 [lbs of B]. Not the intended 10 [lbs of A].

Similarly, mole fraction, volume fraction, and weight fraction are not dimensionless. The ratios represent the fraction of A in the mixture. Even though we consider the fraction to be dimensionless because it is the same measure of quantity for one component in the numerator as it is for the total mixture in the denominator (moles to moles, volume to volume, etc.) it is not dimensionless. Mole fraction is the moles of A per moles of total.

Dimensionless groups are unitless, but truly not dimensionless. Reynolds number again, $Re = \frac{du\rho}{\mu}$ is unitless, but represents separate numerator and denominator phenomena – the rate of momentum conveyed by the flowing fluid along the flow direction to the rate of momentum diffusing perpendicular to the flow direction. Similarly, the ratio of activation energy to thermal energy, $\frac{E}{RT}$, used in reaction kinetics, and vapor-liquid equilibrium is truly not dimensionless it is the activation energy to cause a reaction divided by the average thermal energy in the molecules. But, we consider such ratios to be dimensionless groups, and use them as dimensionless variables in correlation equations and exponentials.

% CV is not the same as % MV. Even both are % of full scale. Conventionally, controller gain has the dimensions of %MV/%CV. Gain multiplies the %CV to convert it to %MV. If the descriptor of the variable label is omitted, then controller gain is %/% which is often considered dimensionless. If gain were a dimensionless number, multiplying it by %CV would return %CV, not %MV. For engineering purposes, for utility and effectiveness, when used properly in the calculation, controller gain can be considered dimensionless.

We use ideal relations (like the Bernoulli relation with the ideal 2 exponent) for pressure loss relations. Then correct it with an adjustable drag coefficient or friction factor. It would be much simpler to use the experimentally determined power of about 1.852 as in the Hazen-Williams relation for turbulent flow friction losses in pipe.

$$\Delta P = (4.52 L Q^{1.852})/(C^{1.852} d^{4.8704})$$

But the 4.52 coefficient is only right if the length is in feet, flow rate is gpm, the pipe roughness factor is from the table relating to the fluid and the pipe, diameter is in inches, and pressure loss is in psi. The 4.52 coefficient is not dimensionless!

Conclusion:

The engineering community uses many conveniences. That we can omit dimensions does not mean a number is dimensionless. That we use equations with missing terms of unity value, does not mean that they can be omitted in a calculation.

For me, precision in terminology is important. I respect the practice utility and unencumbered use of conversions without tracking their dimensional units, but I think the truth about a number needs to be preserved – the convenience of not using dimensions should be explained.

Use such conveniences, but do not let them lead to errors in your analysis.

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